Constructing mathematical arguments using definitions with precision in middle-grades teacher education in the USA

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Limited time to satisfy competing demands in teacher education

We want mathematics teachers to:

- have opportunities to learn mathematical forms of argumentation, including using definitions in constructing arguments (CCSS; Common Core State Standards Initiative, 2010);

- study the mathematics they will teach in depth, from the perspective of a teacher (Conference Board of the Mathematical Sciences, 2012).

Can we satisfy the former within the latter?

Opportunities in the multiplicative conceptual field

The multiplicative conceptual field (e.g., Vergnaud 1988) encompasses multiplication, division, fraction, ratio, and proportional relationships and is a foundation for critical topics including linear functions, rates of change, and slope.

A definition of multiplication in terms of quantities

Beckmann & Izsák, 2015

$$M \cdot N = P$$

$$(\#\text{ of groups}) \cdot (\ \#\text{ of units in 1 group}\ ) = (\ \#\text{ of units in } M \text{ groups}\ )$$

Fertilizer problem

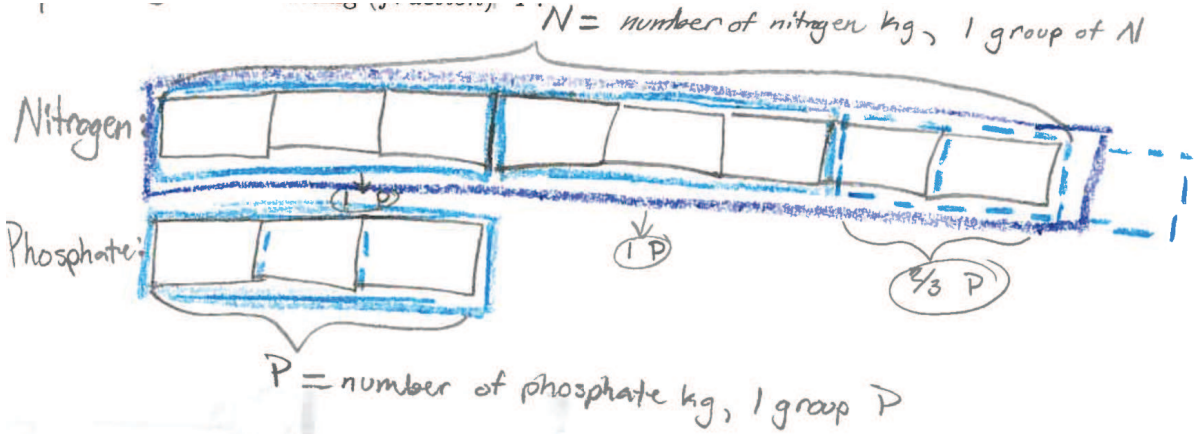
Derive and explain an equation in two variables

A type of fertilizer is made by mixing nitrogen and phosphate in an 8 to 3 ratio.
Suppose you will use
N kilograms of nitrogen and P kilograms of phosphate, . . .
derive and explain an equation of the form

$$(\mathit{fraction}) \cdot P = N$$

. . .

How-many-total-amounts method



? · P = N is asking how much of group P is equal to 1 group of N. With my parts lined up as pictured above, I can see that 2 whole groups of P and an additional 2/3 group of P "fits" or makes up 1 group of N. So,

$$2\tfrac{2}{3} \cdot P = N$$

$2\tfrac{2}{3}$: # groups P; P: size of 1 group P; N: 1 group N

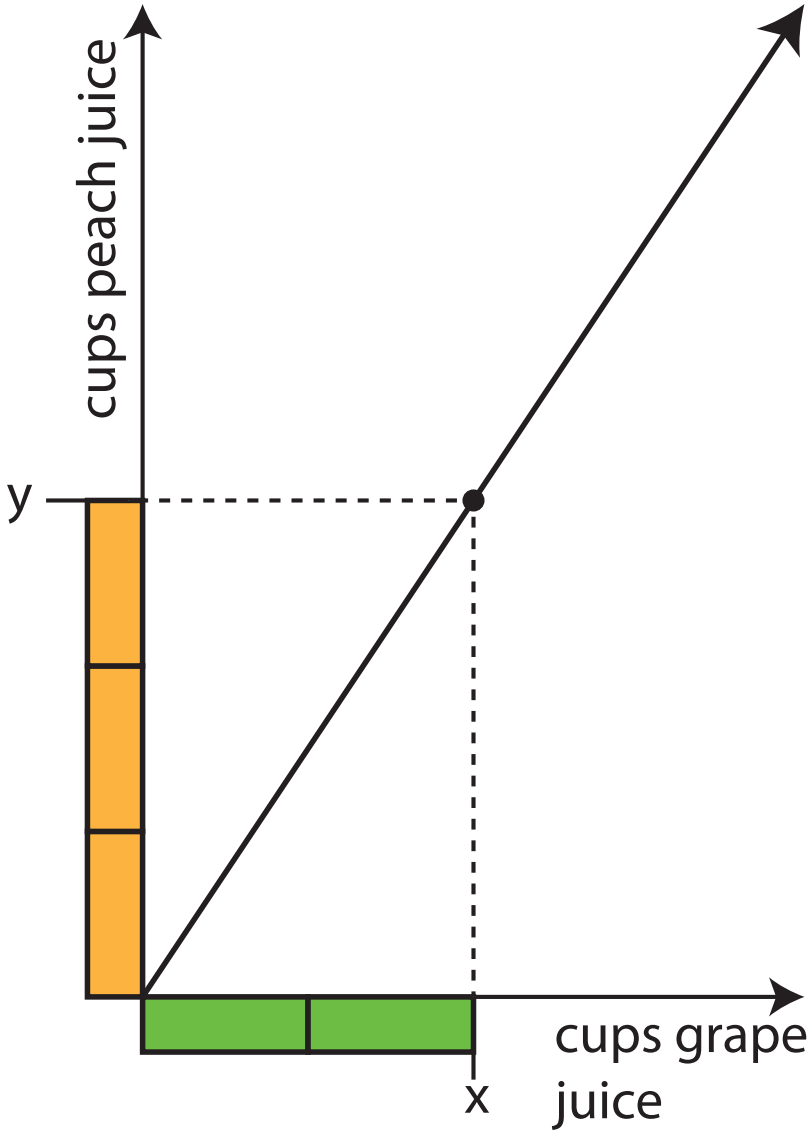
Connections

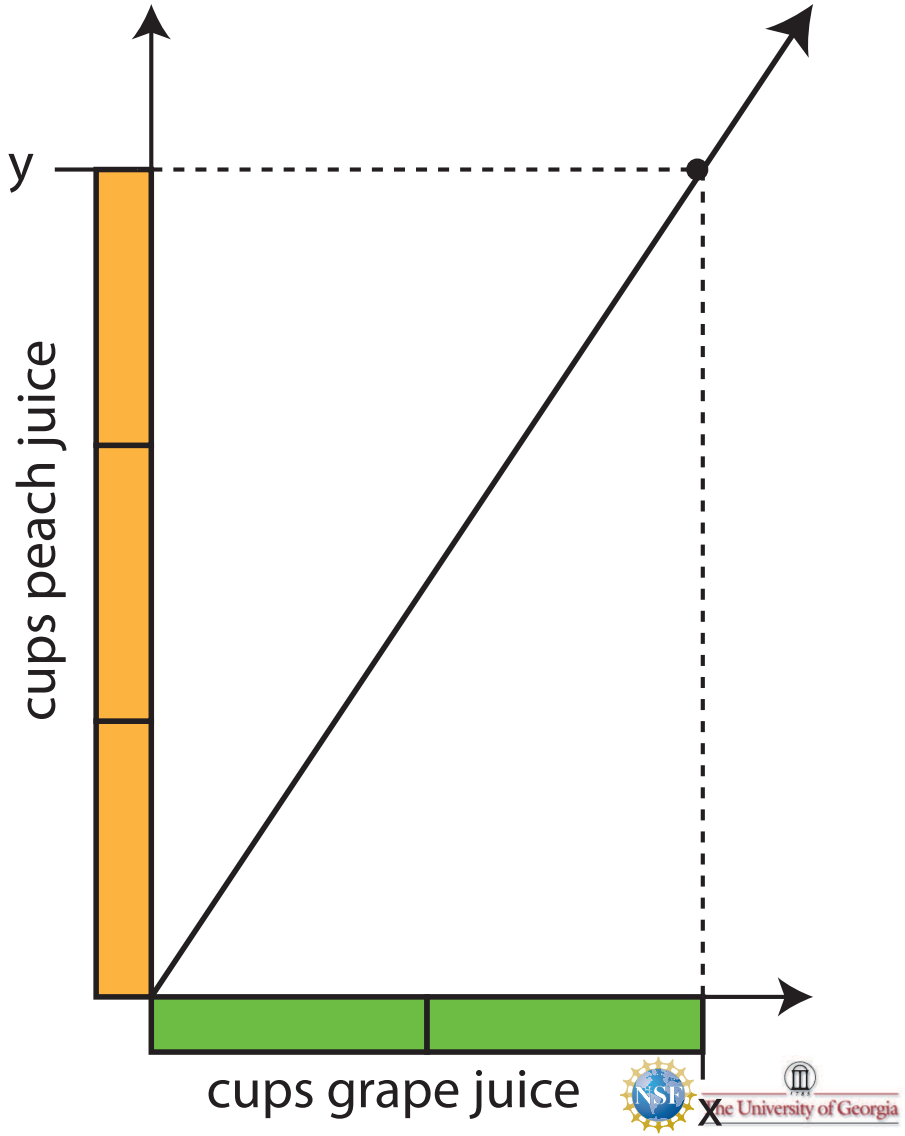
To Kaur's paper: The solution method just presented and another solution method presented in Beckmann, Izsák, and Ölmez's paper use the Model Method.

To Venkat's paper:
"attention to representational competence can provide a bridge that allows for concurrent attention to teachers' learning of mathematics and their teaching of mathematics" (p. 587).

To further mathematics: the case of slope.

Another view of slope (Beckmann & Izsák, 2014)





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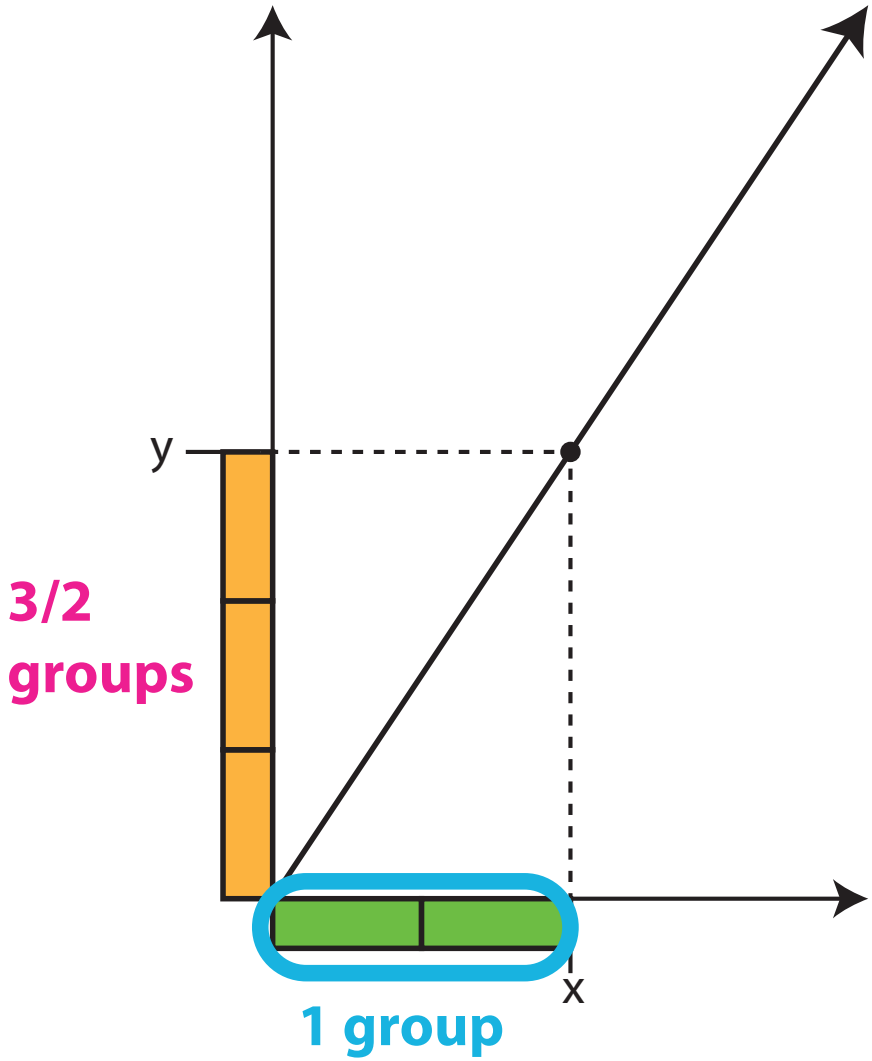

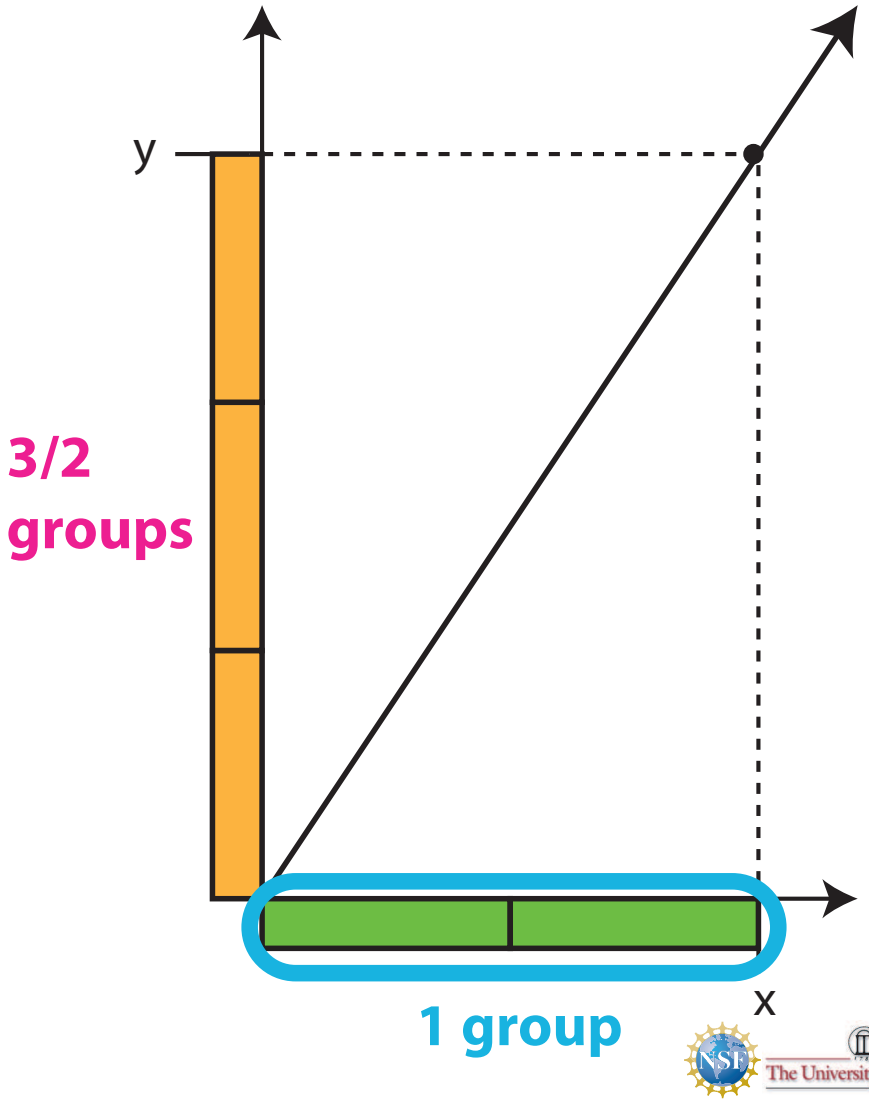

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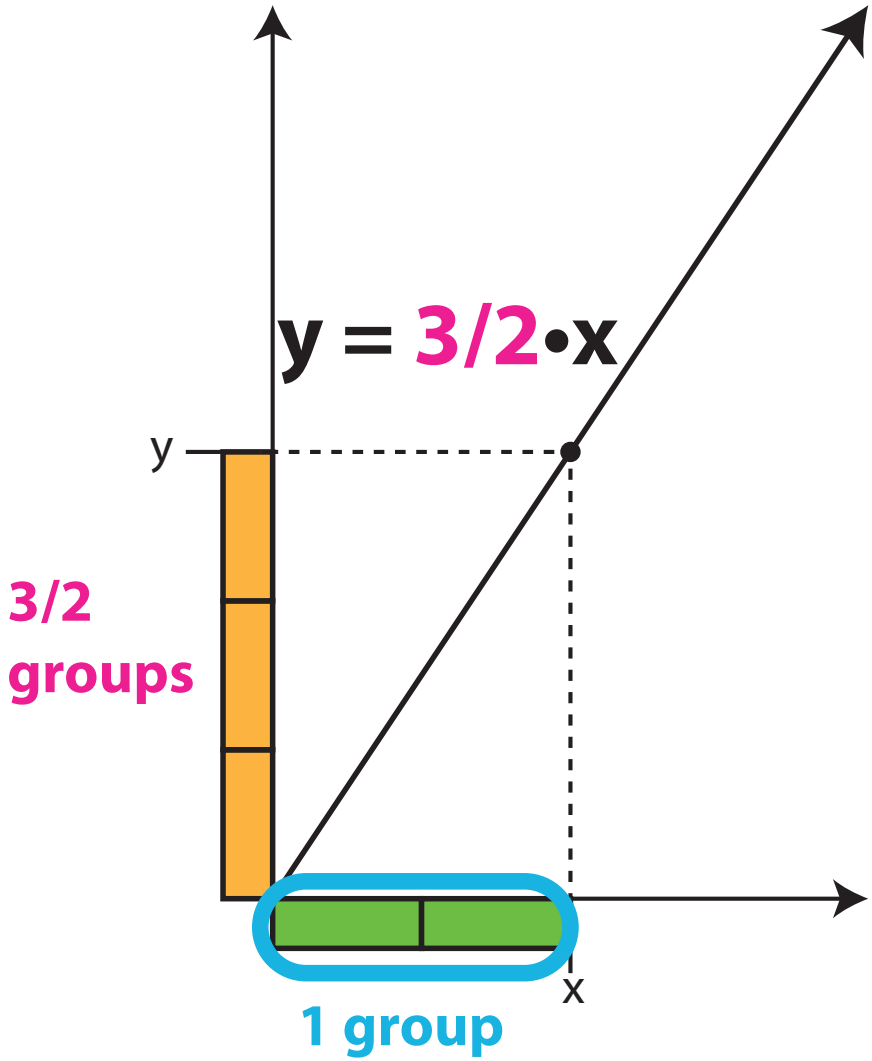

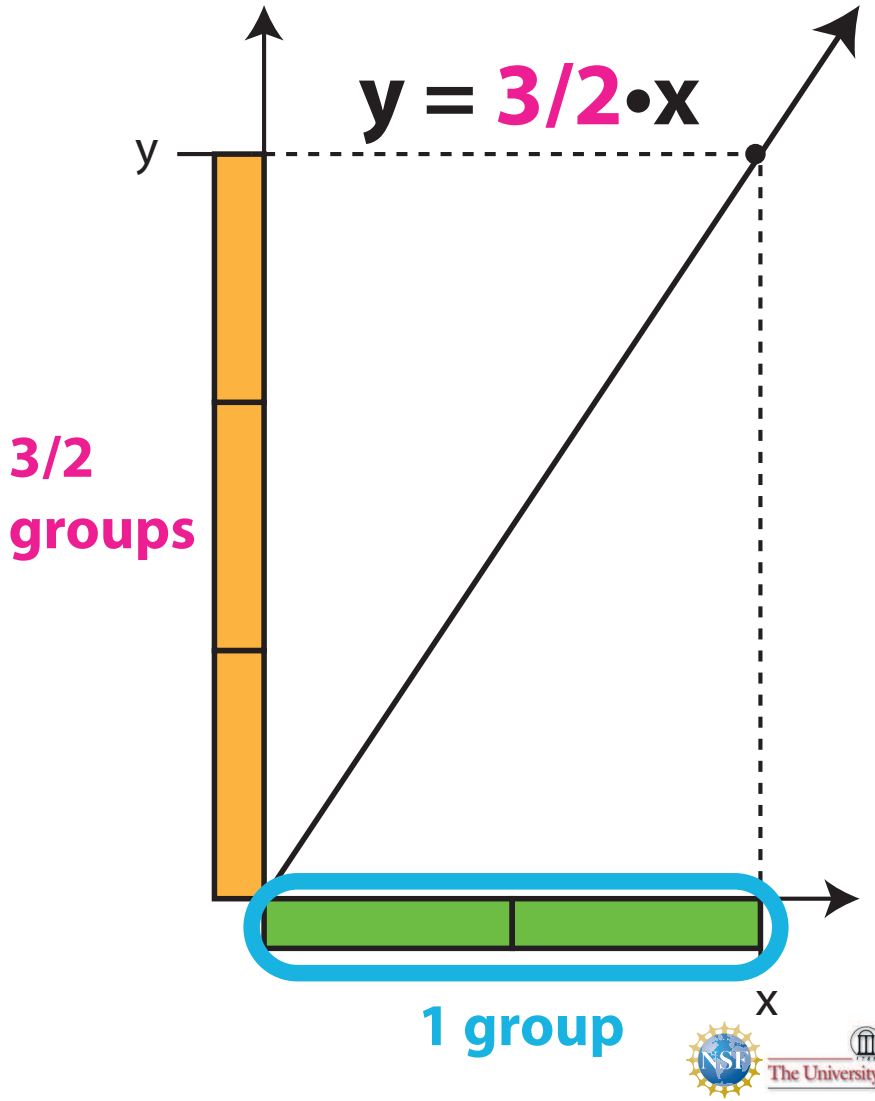

Conclusion

Using a definition of multiplication provides opportunities to

- build skill in constructing viable mathematical arguments;
- deepen understanding of the mathematics teachers will teach;
- lay a foundation for understanding slope, rate of change, equations and functions.

References

Beckmann, S. & Izsák, A. (2014). Variable parts: A new perspective on proportional relationships and linear functions. In Nicol, C., Liljedahl, P., Oesterle, S., & Allan, D. (Eds.) Proceedings of the Joint Meeting of Thirty-Eighth Conference of the International meeting of the Psychology of Mathematics Education and the Thirty-Sixth meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2, pp. 113-120. Vancouver, Canada: PME.

Beckmann, S., & Izsák, A. (2015). Two perspectives on proportional relationships: Extending complementary origins of multiplication in terms of quantities. *Journal for Research in Mathematics Education, 46*(1), pp. 17–38. doi: 10.5951/jresematheduc.46.1.0017.

References

Common Core State Standards Initiative (2010). *The common core state standards for mathematics.* Washington, D.C.: Author.

Conference Board of the Mathematical Sciences (2012). *The Mathematical Education of Teachers II.* Washington, DC: Author.

Vergnaud, G. (1988). Multiplicative Structures. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in middle grades* (pp. 141Ð161). Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Erlbaum.